# Test Statistics

This document provides definitions and some results for tests that detect the presence of a condition (a test result is either “positive” or “negative”, which may be “true” or “false”).

**Definition 1.** A **true positive** test result is one that detects the condition when the condition is present.

**Definition 2.** A **true negative** test result is one that does not detect the condition when the condition is absent.

**Definition 3.** A **false positive** test result is one that detects the condition when the condition is absent.

**Definition 4.** A **false negative** test result is one that does not detect the condition when the condition is present.

| | | condition | |
|---|---|---|---|
| | | present | absent |
| Test | positive | true positive | false positive |
| | negative | false negative | true negative |

Let TP denote the number of true positives, TN the number of true negatives, FP the number of false positives, and FN the number of false negatives.

**Definition 5. Sensitivity** measures the ability of a test to detect the condition when the condition is present. Thus, Sensitivity = TP/(TP+FN).

**Definition 6. Specificity** measures the ability of a test to correctly exclude the condition (not detect the condition) when the condition is absent. Thus, Specificity = TN/(TN+FP).

**Definition 7. Predictive value positive** is the proportion of positives that correspond to the presence of the condition. Thus, Predictive value positive = TP/(TP+FP).

**Definition 8**. **Predictive value negative** is the proportion of negatives that correspond to the absence of the condition. Thus, Predictive value negative = TN/(TN+FN).

| | | Condition | |
|---|---|---|---|
| | | present | Absent |
| test | positive | True positive | False positive |
| | negative | False negative | True negative |

Sensitivity

| | | condition | |
|---|---|---|---|
| | | Present | absent |
| test | Positive | True positive | false positive |
| | negative | False negative | true negative |

Specificity

| | | Condition | |
|---|---|---|---|
| | | present | Absent |
| test | positive | True positive | False positive |
| | negative | False negative | True negative |

Predictive value positive

| | | condition | |
|---|---|---|---|
| | | Present | absent |
| test | positive | True positive | false positive |
| | negative | False negative | true negative |

Predictive value negative

**Definition 9**. **A parallel combination of tests** is a test that is positive if at least one of the combining tests is positive; otherwise it is negative (alternative definition: a parallel combination of tests is a test that is negative only if all the combining tests are negative; otherwise it is positive).

**Definition 10**. **A series combination of tests** is a test that is negative if at least one of the combining tests is negative; otherwise it is positive (alternative definition: a series combination of tests is a test that is positive only if all the combining tests are positive; otherwise it is negative).

**Definition 10**. Two (or more) tests are **independent** if the result of any of the tests does not depend on the result of the other test(s).

**Result 1.** The sensitivity ($SEN$) and specificity ($SPE$) of a parallel combination of $N$ independent tests are

$$SEN = 1 - (1 - SEN_1) \cdot (1 - SEN_2) \cdots (1 - SEN_N),$$

$$SPE = SPE_1 \cdot SPE_2 \cdots SPE_N,$$

respectively, where $SEN_i$ is the sensitivity and $SPE_i$ the specificity of the $i$-th combining test.

**Comment:** It is not difficult to show that $SEN \geq SEN_i$ and $SPE \leq SPE_i$ for $i = 1, \ldots, N$. Thus, a parallel combination of tests increases the sensitivity and decreases the specificity.

**Result 2.** The sensitivity ($SEN$) and specificity ($SPE$) of a series combination of $N$ independent tests are

$$SEN = SEN_1 \cdot SEN_2 \cdots SEN_N,$$

$$SPE = 1 - (1 - SPE_1) \cdot (1 - SPE_2) \cdots (1 - SPE_N),$$

respectively, where $SEN_i$ is the sensitivity and $SPE_i$ the specificity of the $i$-th combining test.

**Comment:** It is not difficult to show that $SEN \leq SEN_i$ and $SPE \geq SPE_i$ for $i = 1, \ldots, N$. Thus, a series combination of tests decreases the sensitivity and increases the specificity.