PROBLEMS

1. The following image contains two regions that are not necessarily connected,

\[
\begin{bmatrix}
3 & 4 & 3 & 2 \\
9 & 7 & 9 & 4 \\
8 & 8 & 3 & 4 \\
2 & 3 & 2 & 10
\end{bmatrix}
\]

a. Determine the global threshold for segmenting the image using the iterative procedure from Gonzalez and Woods, pages 599-600. Assume that \( T = 0 \) initially. Show the two means and the threshold for each iteration.

b. Sketch the histogram of the image. Does the computed global threshold make sense? Briefly explain.

c. Segment the image using the global threshold computed in a. The pixels with values greater or equal to the threshold set to 1. The rest of the pixels set to 0.

2. A continuous image contains an object that occupies 40% of the image. The rest of the image represents the background. The gray level intensity over the object is 80 and over the background it is 40. The image is corrupted with additive noise that has the following distribution

\[
p(r) = \frac{\sqrt{a}}{\pi} \frac{1}{1 + a(r - \mu)^2},
\]

where \( \mu \) is the mean and \( a > 0 \) is the second parameter of the distribution. The noise is zero mean and it is added to the original image. It is known that \( a = 3 \) over the background, and \( a = 2 \) over the object. Thus, the distribution of gray level intensities over the background is

\[
p_1(r) = \frac{\sqrt{3}}{\pi} \frac{1}{1 + 3(r - 40)^2},
\]

while over the object it is

\[
p_2(r) = \frac{\sqrt{2}}{\pi} \frac{1}{1 + 2(r - 80)^2}.
\]

a. Write the expression for the distribution of gray level intensities (normalized histogram) over the entire image. Sketch the distribution.
b. Since the two modes of the distribution overlap, one cannot find a global threshold value that would ideally segment the object, i.e. ideally separate the object from the background. For a certain threshold value, a part of the object will be mistakenly segmented as the background, and vice-versa, i.e. there will always be an error. Your goal is to compute the optimal global threshold (a value between 40 and 80) that would minimize the segmentation error.

SOLUTIONS

1.

a. 
Iteration 1: \( T_0 = 0 \quad \Rightarrow \quad \mu_1 = 0 \quad \mu_2 = 5.06 \quad \Rightarrow \quad T_1 = \frac{\mu_1 + \mu_2}{2} = 2.53 \)

Iteration 2: \( T_1 = 2.53 \quad \Rightarrow \quad \mu_1 = 2 \quad \mu_2 = 5.77 \quad \Rightarrow \quad T_2 = \frac{\mu_1 + \mu_2}{2} = 3.88 \)

Iteration 3: \( T_2 = 3.88 \quad \Rightarrow \quad \mu_1 = 2.57 \quad \mu_2 = 7 \quad \Rightarrow \quad T_3 = \frac{\mu_1 + \mu_2}{2} = 4.79 \)

Iteration 4: \( T_3 = 4.79 \quad \Rightarrow \quad \mu_1 = 3 \quad \mu_2 = 8.5 \quad \Rightarrow \quad T_4 = \frac{\mu_1 + \mu_2}{2} = 5.75 \)

Iteration 5: \( T_4 = 5.75 \quad \Rightarrow \quad \mu_1 = 3 \quad \mu_2 = 8.5 \quad \Rightarrow \quad T_5 = \frac{\mu_1 + \mu_2}{2} = 5.75 \)

Since \( T_4 = T_5 \), the iterative procedure stops and the global threshold is \( T = 5.75 \).

b. The histogram of the image is given in the following figure.
The histogram has two modes that are clearly separated. The computed global threshold falls into the gap between the two modes and ideally segments the image into the two regions.

c. The segmented image is

\[
\begin{bmatrix}
0 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 \\
1 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

2.

a. The probability of points belonging to the background is \( P_1 = .6 \), and the probability of the points belonging to the object is \( P_2 = .4 \). The distribution of gray level intensities for the entire image is

\[
p(r) = P_1 p_1(r) + P_2 p_2(r) = \\
\frac{.6 \sqrt{3}}{\pi} \frac{1}{1 + 3(r - 40)^2} + \frac{.4 \sqrt{2}}{\pi} \frac{1}{1 + 2(r - 80)^2}.
\]
The graph of the distribution is shown in the following figure.

\[ \frac{.6 \sqrt{3}}{\pi} \frac{1}{1 + 3(T - 40)^2} = \frac{.4 \sqrt{2}}{\pi} \frac{1}{1 + 2(T - 80)^2}. \]

By rearranging the terms, one can see that the above equation reduces to the following quadratic equation,

\[ T^2 - 515.965T + 27759.135 = 0. \]

The solution of the equation that is between 40 and 80 is \( T = 61 \). Thus, a global threshold of 61 minimizes the segmentation error.