Sampling and Quantization

BMED/ECE 4783
Introduction to Medical Image Processing
Fall 2009

Lecture Outline

- Sampling
- Quantization
- Sampling theorem
- Interpolation

Generating a digital image

Coordinate Convention

Storage Space Requirement
Subsampling

Figure 2.19: A 3024 x 3024 8-bit image subsampled down to 32 x 32 pixels. The number of allowable gray levels was kept at 256.

Gonzalez and Woods, Digital Image Processing

Resampling

Figure 2.20: (a) 1024 x 1024 8-bit image. (b) 128 x 128 image subsampled to 128 x 128 pixels for use as a zoom and down-sampling example. (c) Through (f): 256 x 256, 128 x 128, 64 x 64, and 32 x 32 images interpolated and resampled to 128 x 128 pixels.

Gonzalez and Woods, Digital Image Processing

Varying the Number of Gray Levels

Figure 2.21: (a) Original image. (b) Image enlarged by 1/8 and resampled by 1/8, preserving the original resolution and smoothing the resolution transition.

Gonzalez and Woods, Digital Image Processing

Varying the Number of Gray Levels

Figure 2.22: (a) Original image. (b) Image enlarged by 1/8 and resampled by 1/8, preserving the original resolution and smoothing the resolution transition.

Gonzalez and Woods, Digital Image Processing

Image Interpolation

Figure 2.23: Top row: Images resampled from 128 x 128, 64 x 64, and 32 x 32 pixels to 1024 x 1024 pixels using nearest neighbor gray-level interpolation. Bottom row: Same sequence but using bilinear interpolation.

Gonzalez and Woods, Digital Image Processing

Sampling

• Given a function f(x,y), how many samples per unit area is the “optimum”? 
• What is the “optimum”? 
• E.g. f(x,y) is a gray level image, i.e. f(x,y) represents the gray level intensity at the location (x,y).
• E.g. if x and y are expressed in millimeters (mm), the corresponding unit area is mm^2. How many samples per mm^2 is the optimum? 5 samples/mm^2 ? 1,000,000,000 samples/mm^2?
Sampling

- An image (function \( f(x,y) \)) is sampled in order to be digitally stored, transmitted, and processed.
- At some point the image has to be reproduced from samples, i.e. converted back to analog form (e.g. to be displayed). This is digital-to-analog conversion (DAC)
- Too few samples \( \Rightarrow \) low quality of the reproduced image
- Too many samples \( \Rightarrow \) too much memory to store the image (i.e. samples), too much time to process the image (samples), too much time to transmit the image (samples)
- Optimal number of samples: the minimal number of samples that allows for exact reconstruction of the image

Band-limited images (2D functions)

An image is band-limited if its spectrum has a bounded region of support, i.e. if

\[
F(u, v) = 0 \quad \text{when} \quad |u| > u_{\text{max}} \quad \text{or} \quad |v| > v_{\text{max}}
\]

\( u_{\text{max}} \) and \( v_{\text{max}} \) are the maximal frequencies present in \( f(x,y) \)

- Real images are band-limited
- Often, the maximal considered frequencies are determined not by the images themselves, but by other factors (e.g. human vision is band-limited; we cannot see details smaller than something)

2D Sampling Theorem

A band-limited function \( f(x,y) \) can be exactly reconstructed from its samples if it is sampled with sampling frequencies \( u' \) and \( v' \) that satisfy the following:

\[
u' > 2u_{\text{max}} \quad \text{and} \quad v' > 2v_{\text{max}}
\]

where \( u_{\text{max}} \) and \( v_{\text{max}} \) are the maximal frequencies present in \( f(x,y) \).

\[
\Delta_x = \frac{1}{u'} \quad \text{and} \quad \Delta_y = \frac{1}{v'}
\]

\( \Delta_x, \Delta_y \) - sampling intervals

Reconstruction from Samples

Goal: given the sampled function \( f_s(x,y) \), reconstruct \( f(x,y) \)

The reconstructed function is denoted by \( f_r(x,y) \)

\[
F_r(u, v) = F_s(u, v)R(u, v)
\]

\[
R(u, v) = \begin{cases} 
K, & |u| \leq u_c \text{ and } |v| \leq v_c \\
0, & \text{otherwise}
\end{cases}
\]

\( K \) - scaling constant \( u_c, v_c \) - cutoff frequencies

Reconstruction from Samples

If \( f(x,y) \) is band-limited, the sampling theorem is satisfied, and

\[
u_{\text{max}} \leq u_c \leq u' - u_{\text{max}} \quad \text{and} \quad v_{\text{max}} \leq v_c \leq v' - v_{\text{max}}
\]

then the function is reconstructed exactly, i.e. \( f_r(x,y) = f(x,y) \).

If the reconstruction is not exact, we say that there is aliasing.